# 5 Systems of Linear Equations

"Can you graph a system of linear equations that shows the number of biscuits and treats that I am going to share with you?"

"Hey, look over here. Can you estimate the solution of the system of linear equations that I made with these cattails?"

# What You Learned Before


"Hold your tail a bit lower."

## Combining Like Terms (6.EE.3)

**Example 1** Simplify each expression.

**a.** $4x + 7 + 5x - 2$

$$
\begin{aligned}
4x + 7 + 5x - 2 &= 4x + 5x + 7 - 2 && \text{Commutative Property of Addition} \\
&= (4 + 5)x + 7 - 2 && \text{Distributive Property} \\
&= 9x + 5 && \text{Simplify.}
\end{aligned}
$$

**b.** $z + z + z + z$

$$
\begin{aligned}
z + z + z + z &= 1z + 1z + 1z + 1z && \text{Multiplication Property of One} \\
&= (1 + 1 + 1 + 1)z && \text{Distributive Property} \\
&= 4z && \text{Add coefficients.}
\end{aligned}
$$

### Try It Yourself

Simplify the expression.

**1.** $5 + 4z - 2z$

**2.** $5(c + 8) + c + 3$

## Solving Multi-Step Equations (8.EE.7b)

**Example 2** Solve $4x - 2(3x + 1) = 16$.

$$
\begin{aligned}
4x - 2(3x + 1) &= 16 && \text{Write the equation.} \\
4x - 6x - 2 &= 16 && \text{Distributive Property} \\
-2x - 2 &= 16 && \text{Combine like terms.} \\
-2x &= 18 && \text{Add 2 to each side.} \\
x &= -9 && \text{Divide each side by } -2.
\end{aligned}
$$

The solution is $x = -9$.

### Try It Yourself

Solve the equation. Check your solution.

**3.** $-5x + 8 = -7$

**4.** $7w + w - 15 = 17$

**5.** $-3(z - 8) + 10 = -5$

**6.** $2 = 10c - 4(2c - 9)$

**Essential Question** How can you solve a system of linear equations?

### 1 ACTIVITY: Writing a System of Linear Equations

**Work with a partner.**

**Your family starts a bed-and-breakfast. It spends $500 fixing up a bedroom to rent. The cost for food and utilities is $10 per night. Your family charges $60 per night to rent the bedroom.**

a. Write an equation that represents the costs.

| Cost, $C$ (in dollars) | = | $10 per night | · | Number of nights, $x$ | + | $500 |
|---|---|---|---|---|---|---|

b. Write an equation that represents the revenue (income).

| Revenue, $R$ (in dollars) | = | $60 per night | · | Number of nights, $x$ |
|---|---|---|---|---|

c. A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

### 2 ACTIVITY: Using a Table to Solve a System

**Work with a partner. Use the cost and revenue equations from Activity 1 to find how many nights your family needs to rent the bedroom before recovering the cost of fixing up the bedroom. This is the *break-even point*.**

a. Copy and complete the table.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | | | | | | | | | | | | |
| $R$ | | | | | | | | | | | | |

b. How many nights does your family need to rent the bedroom before breaking even?

## 3    ACTIVITY: Using a Graph to Solve a System

**Work with a partner.**

**a.** Graph the cost equation from Activity 1.

**b.** In the same coordinate plane, graph the revenue equation from Activity 1.

**c.** Find the point of intersection of the two graphs. What does this point represent? How does this compare to the break-even point in Activity 2? Explain.
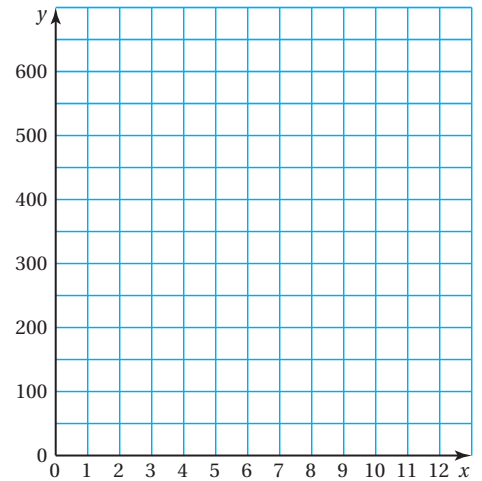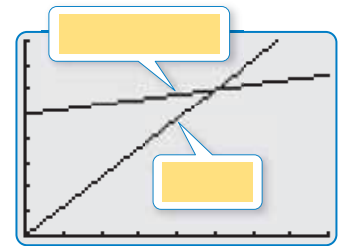
## 4    ACTIVITY: Using a Graphing Calculator

**Work with a partner. Use a graphing calculator to solve the system.**

$y = 10x + 500$    Equation 1
$y = 60x$    Equation 2

**Math Practice 5**

**Use Technology to Explore**

How do you decide the values for the viewing window of your calculator? What other viewing windows could you use?

**a.** Enter the equations into your calculator. Then graph the equations. What is an appropriate window?

**b.** On your graph, how can you determine which line is the graph of which equation? Label the equations on the graph shown.

**c.** Visually estimate the point of intersection of the graphs.

**d.** To find the solution, use the *intersect* feature to find the point of intersection. The solution is ( ⬜ , ⬜ ).

# What Is Your Answer?

**5.** **IN YOUR OWN WORDS** How can you solve a system of linear equations? How can you check your solution?

**6.** **CHOOSE TOOLS** Solve one of the systems by using a table, another system by sketching a graph, and the remaining system by using a graphing calculator. Explain why you chose each method.

**a.** $y = 4.3x + 1.2$
$y = -1.7x - 2.4$

**b.** $y = x$
$y = -2x + 9$

**c.** $y = -x - 5$
$y = 3x + 1$

**Practice**

Use what you learned about systems of linear equations to complete Exercises 4–6 on page 206.

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**Key Vocabulary**
system of linear equations, *p. 204*
solution of a system of linear equations, *p. 204*

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown below.

$$y = x + 1 \qquad \text{Equation 1}$$
$$y = 2x - 7 \qquad \text{Equation 2}$$

A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system. The solution of a system of linear equations is the point of intersection of the graphs of the equations.

## Key Idea

**Reading**
A system of linear equations is also called a *linear system*.

**Solving a System of Linear Equations by Graphing**

**Step 1:** Graph each equation in the same coordinate plane.

**Step 2:** Estimate the point of intersection.

**Step 3:** Check the point from Step 2 by substituting for $x$ and $y$ in each equation of the original system.

## EXAMPLE 1 Solving a System of Linear Equations by Graphing
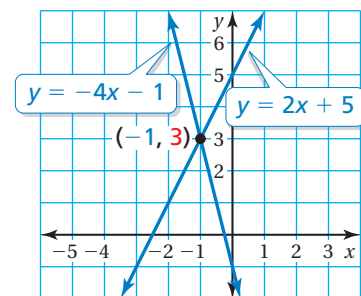
**Solve the system by graphing.** $\quad y = 2x + 5 \qquad$ Equation 1
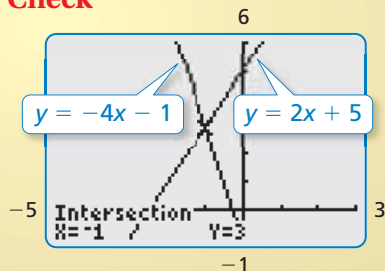$$y = -4x - 1 \qquad \text{Equation 2}$$

**Step 1:** Graph each equation.

**Step 2:** Estimate the point of intersection. The graphs appear to intersect at $(-1, 3)$.

**Step 3:** Check the point from Step 2.

**Check**

| Equation 1 | Equation 2 |
|---|---|
| $y = 2x + 5$ | $y = -4x - 1$ |
| $3 \overset{?}{=} 2(-1) + 5$ | $3 \overset{?}{=} -4(-1) - 1$ |
| $3 = 3$ ✓ | $3 = 3$ ✓ |

The solution is $(-1, 3)$.

## On Your Own

**Now You're Ready**
Exercises 10−12

**Solve the system of linear equations by graphing.**

**1.** $y = x - 1$
$\quad\ y = -x + 3$

**2.** $y = -5x + 14$
$\quad\ y = x - 10$

**3.** $y = x$
$\quad\ y = 2x + 1$

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**EXAMPLE** 2 **Real-Life Application**

A kicker on a football team scores 1 point for making an extra point and 3 points for making a field goal. The kicker makes a total of 8 extra points and field goals in a game and scores 12 points. Write and solve a system of linear equations to find the number $x$ of extra points and the number $y$ of field goals.
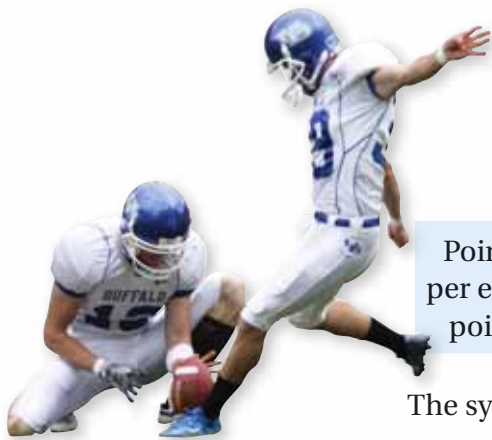
Use a verbal model to write a system of linear equations.

| | | | | |
|---|---|---|---|---|
| Number of extra points, $x$ | $+$ | Number of field goals, $y$ | $=$ | Total number of kicks |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Points per extra point | $\cdot$ | Number of extra points, $x$ | $+$ | Points per field goal | $\cdot$ | Number of field goals, $y$ | $=$ | Total number of points |

The system is:  $x + y = 8$  **Equation 1**

$x + 3y = 12$  **Equation 2**

**Step 1:** Graph each equation.

**Step 2:** Estimate the point of intersection. The graphs appear to intersect at (6, 2).

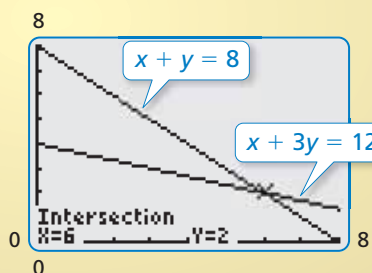**Step 3:** Check your point from Step 2.
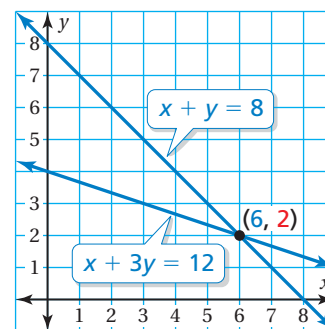
**Study Tip**

It may be easier to graph the equations in a system by rewriting the equations in slope-intercept form.

| **Equation 1** | **Equation 2** |
|---|---|
| $x + y = 8$ | $x + 3y = 12$ |
| $6 + 2 \overset{?}{=} 8$ | $6 + 3(2) \overset{?}{=} 12$ |
| $8 = 8$ ✓ | $12 = 12$ ✓ |

The solution is (6, 2). So, the kicker made 6 extra points and 2 field goals.

**Check**

## On Your Own

Solve the system of linear equations by graphing.

**4.** $y = -4x - 7$

$x + y = 2$

**5.** $x - y = 5$

$-3x + y = -1$

**6.** $\dfrac{1}{2}x + y = -6$

$6x + 2y = 8$

**7. WHAT IF?** The kicker makes a total of 7 extra points and field goals and scores 17 points. Write and solve a system of linear equations to find the numbers of extra points and field goals.

## ✓ Vocabulary and Concept Check

1. **VOCABULARY** Do the equations $4x - 3y = 5$ and $7y + 2x = -8$ form a system of linear equations? Explain.

2. **WRITING** What does it mean to solve a system of equations?

3. **WRITING** You graph a system of linear equations, and the solution appears to be (3, 4). How can you verify that the solution is (3, 4)?

## Practice and Problem Solving

**Use a table to find the break-even point. Check your solution.**

4. $C = 15x + 150$
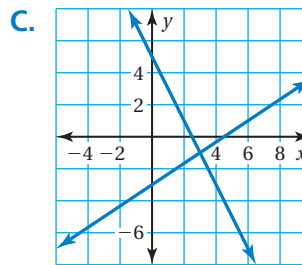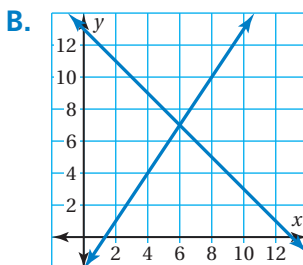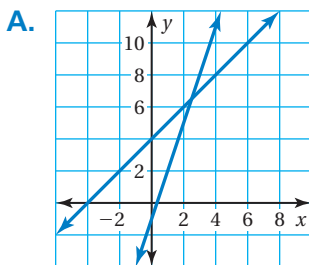   $R = 45x$

5. $C = 24x + 80$
   $R = 44x$

6. $C = 36x + 200$
   $R = 76x$

**Match the system of linear equations with the corresponding graph. Use the graph to estimate the solution. Check your solution.**

7. $y = 1.5x - 2$
   $y = -x + 13$

8. $y = x + 4$
   $y = 3x - 1$

9. $y = \frac{2}{3}x - 3$
   $y = -2x + 5$

A.

B.

C.

**Solve the system of linear equations by graphing.**

❶ 10. $y = 2x + 9$
   $y = 6 - x$

11. $y = -x - 4$
   $y = \frac{3}{5}x + 4$

12. $y = 2x + 5$
   $y = \frac{1}{2}x - 1$

❷ 13. $x + y = 27$
   $y = x + 3$

14. $y - x = 17$
   $y = 4x + 2$

15. $x - y = 7$
   $0.5x + y = 5$

16. **CARRIAGE RIDES** The cost $C$ (in dollars) for the care and maintenance of a horse and carriage is $C = 15x + 2000$, where $x$ is the number of rides.

   a. Write an equation for the revenue $R$ in terms of the number of rides.
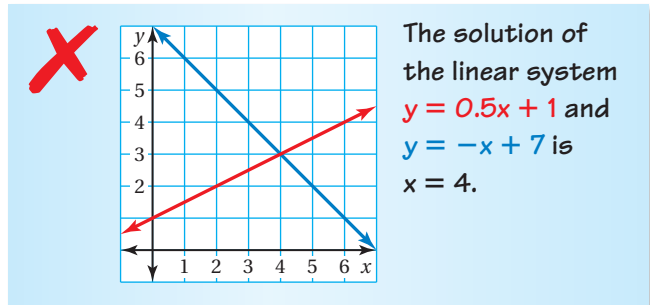
   b. How many rides are needed to break even?

$35 per ride

**Use a graphing calculator to solve the system of linear equations.**

**17.** $2.2x + y = 12.5$
$1.4x - 4y = 1$

**18.** $2.1x + 4.2y = 14.7$
$-5.7x - 1.9y = -11.4$

**19.** $-1.1x - 5.5y = -4.4$
$0.8x - 3.2y = -11.2$

**20. ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

**21. REASONING** Is it possible for a system of two linear equations to have exactly two solutions? Explain your reasoning.



The solution of the linear system $y = 0.5x + 1$ and $y = -x + 7$ is $x = 4$.

**22. MODELING** You have a total of 42 math and science problems for homework. You have 10 more math problems than science problems. How many problems do you have in each subject? Use a system of linear equations to justify your answer.
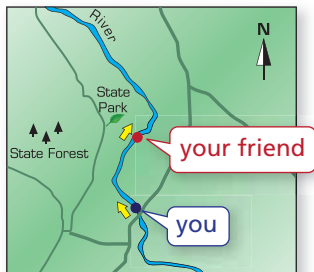


**23. CANOE RACE** You and your friend are in a canoe race. Your friend is a half mile in front of you and paddling 3 miles per hour. You are paddling 3.4 miles per hour.

**a.** You are 8.5 miles from the finish line. How long will it take you to catch up to your friend?

**b.** You both maintain your paddling rates for the remainder of the race. How far ahead of your friend will you be when you cross the finish line?

**24. Critical Thinking** Your friend is trying to grow her hair as long as her cousin's hair. The table shows their hair lengths (in inches) in different months.

| Month | Friend's Hair (in.) | Cousin's Hair (in.) |
|-------|--------------------|--------------------|
| 3 | 4 | 7 |
| 8 | 6.5 | 9 |

**a.** Write a system of linear equations that represents this situation.

**b.** Will your friend's hair ever be as long as her cousin's hair? If so, in what month?

## Fair Game Review *What you learned in previous grades & lessons*

**Solve the equation. Check your solution.** *(Section 1.2)*

**25.** $\dfrac{3}{4}c - \dfrac{1}{4}c + 3 = 7$

**26.** $5(2 - y) + y = -6$

**27.** $6x - 3(x + 8) = 9$

**28. MULTIPLE CHOICE** What is the slope of the line that passes through $(-2, -2)$ and $(3, -1)$? *(Section 4.2)*

**(A)** $-5$

**(B)** $-\dfrac{1}{5}$

**(C)** $\dfrac{1}{5}$

**(D)** $5$

# 5.2 Solving Systems of Linear Equations by Substitution

## Essential Question How can you use substitution to solve a system of linear equations?

### 1 ACTIVITY: Using Substitution to Solve a System

**Work with a partner. Solve each system of linear equations by using two methods.**

$$y = 6x - 11$$

**Method 1:** Solve for $x$ first.

Solve for $x$ in one of the equations. Use the expression for $x$ to find the solution of the system. Explain how you did it.

**Method 2:** Solve for $y$ first.

Solve for $y$ in one of the equations. Use the expression for $y$ to find the solution of the system. Explain how you did it.

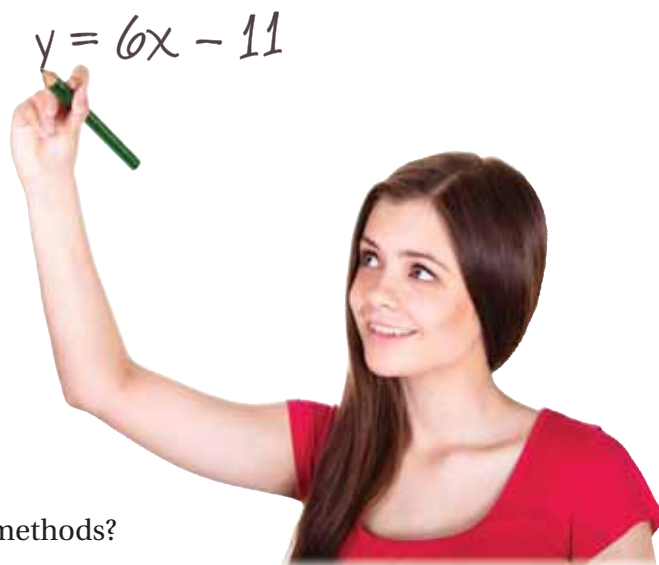Is the solution the same using both methods?

**a.** $6x - y = 11$
$2x + 3y = 7$

**b.** $2x - 3y = -1$
$x - y = 1$

**c.** $3x + y = 5$
$5x - 4y = -3$

**d.** $5x - y = 2$
$3x - 6y = 12$

**e.** $x + y = -1$
$5x + y = -13$

**f.** $2x - 6y = -6$
$7x - 8y = 5$

### 2 ACTIVITY: Writing and Solving a System of Equations

**Work with a partner.**

**a.** Roll a pair of number cubes that have different colors. Then write the ordered pair shown by the number cubes. The ordered pair at the right is (3, 4).
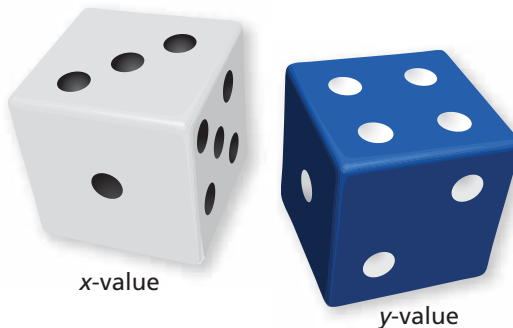
**b.** Write a system of linear equations that has this ordered pair as its solution.

*x*-value

*y*-value

**c.** Exchange systems with your partner. Use one of the methods from Activity 1 to solve the system.

**ACTIVITY:**  Solving a Secret Code

**Check Progress**

As you complete each system of equations, how do you know your answer is correct?

**Work with a partner.  Decode the quote by Archimedes.**



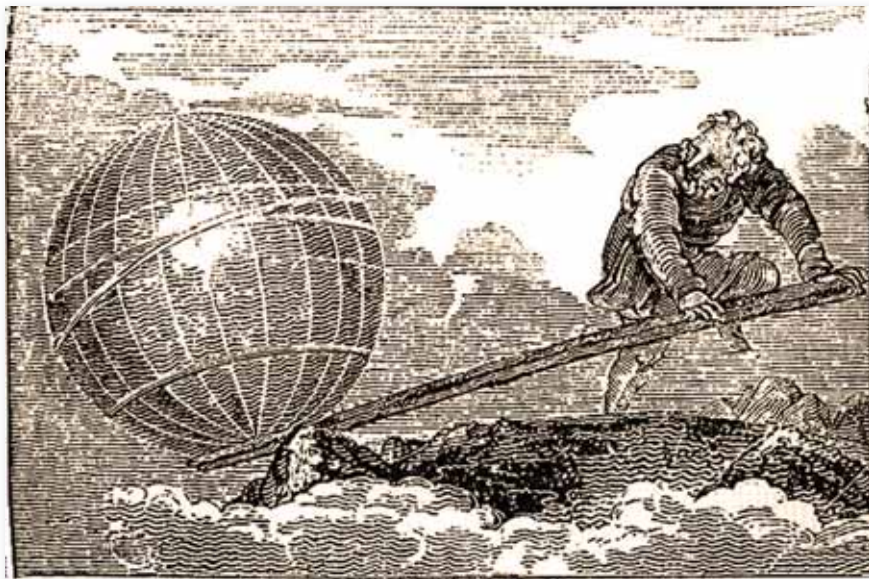$$\overline{\phantom{x}}\,\overline{\phantom{x}}\;\overline{\phantom{x}}\;\overline{\phantom{x}}\quad\overline{\phantom{x}}\,\overline{\phantom{x}}\;\overline{\phantom{x}}\quad\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\quad\overline{\phantom{x}}\,\overline{\phantom{x}}\quad\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,$$
$$-8\;-7\;\;7\;\;-5\;\;\;-4\;-5\;\;\;-3\;\;\;-2\;-1\;-3\;\;0\;\;-5\;\;\;\;1\;\;2\;\;\;\;\;3\;\;\;1\;\;-3\;\;4\;\;\;5\quad,$$

$$\overline{\phantom{x}}\,\overline{\phantom{x}}\;\overline{\phantom{x}}\quad\overline{\phantom{x}}\quad\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\quad\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\quad\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\quad\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,\overline{\phantom{x}}\,$$
$$-3\;\;4\;\;5\;\;\;\;-7\;\;\;\;6\;\;-7\;-1\;-1\;\;\;-4\;\;2\;\;7\;\;-5\;\;\;\;1\;\;8\;\;-5\;\;\;\;-5\;-3\;\;9\;\;1\;\;\;8\quad.$$

| | | | | | |
|---|---|---|---|---|---|
| **(A, C)** | $x + y = -3$ | **(D, E)** | $x + y = 0$ | **(G, H)** | $x + y = 0$ |
| | $x - y = -3$ | | $x - y = 10$ | | $x - y = -16$ |
| **(I, L)** | $x + 2y = -9$ | **(M, N)** | $x + 2y = 4$ | **(O, P)** | $x + 2y = -2$ |
| | $2x - y = -13$ | | $2x - y = -12$ | | $2x - y = 6$ |
| **(R, S)** | $2x + y = 21$ | **(T, U)** | $2x + y = -7$ | **(V, W)** | $2x + y = 20$ |
| | $x - y = 6$ | | $x - y = 10$ | | $x - y = 1$ |

## What Is Your Answer?

**4.** **IN YOUR OWN WORDS**  How can you use substitution to solve a system of linear equations?

**Practice**   Use what you learned about systems of linear equations to complete Exercises 4−6 on page 212.

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Another way to solve systems of linear equations is to use substitution.

## Key Idea

**Solving a System of Linear Equations by Substitution**

**Step 1:** Solve one of the equations for one of the variables.

**Step 2:** Substitute the expression from Step 1 into the other equation and solve for the other variable.

**Step 3:** Substitute the value from Step 2 into one of the original equations and solve.

## EXAMPLE 1 Solving a System of Linear Equations by Substitution

**Solve the system by substitution.**    $y = 2x - 4$    Equation 1

$7x - 2y = 5$    Equation 2

**Step 1:** Equation 1 is already solved for $y$.

**Step 2:** Substitute $2x - 4$ for $y$ in Equation 2.

$$7x - 2y = 5 \qquad \text{Equation 2}$$
$$7x - 2(2x - 4) = 5 \qquad \text{Substitute } 2x - 4 \text{ for } y.$$
$$7x - 4x + 8 = 5 \qquad \text{Distributive Property}$$
$$3x + 8 = 5 \qquad \text{Combine like terms.}$$
$$3x = -3 \qquad \text{Subtract 8 from each side.}$$
$$x = -1 \qquad \text{Divide each side by 3.}$$

**Step 3:** Substitute $-1$ for $x$ in Equation 1 and solve for $y$.

$$y = 2x - 4 \qquad \text{Equation 1}$$
$$= 2(-1) - 4 \qquad \text{Substitute } -1 \text{ for } x.$$
$$= -2 - 4 \qquad \text{Multiply.}$$
$$= -6 \qquad \text{Subtract.}$$

∴ The solution is $(-1, -6)$.

**Check**

Equation 1

$y = 2x - 4$

$-6 \overset{?}{=} 2(-1) - 4$

$-6 = -6$ ✓

Equation 2

$7x - 2y = 5$

$7(-1) - 2(-6) \overset{?}{=} 5$

$5 = 5$ ✓

## On Your Own

Now You're Ready
Exercises 10–15

**Solve the system of linear equations by substitution. Check your solution.**

**1.** $y = 2x + 3$

$y = 5x$

**2.** $4x + 2y = 0$

$y = \dfrac{1}{2}x - 5$

**3.** $x = 5y + 3$

$2x + 4y = -1$

**EXAMPLE** **2**   **Real-Life Application**

**You buy a total of 50 turkey burgers and veggie burgers for $90. You pay $2 per turkey burger and $1.50 per veggie burger. Write and solve a system of linear equations to find the number $x$ of turkey burgers and the number $y$ of veggie burgers you buy.**

Use a verbal model to write a system of linear equations.

| Number of turkey burgers, $x$ | + | Number of veggie burgers, $y$ | = | Total number of burgers |

| Cost per turkey burger | • | Number of turkey burgers, $x$ | + | Cost per veggie burger | • | Number of veggie burgers, $y$ | = | Total cost |

The system is:   $x + y = 50$        Equation 1
                 $2x + 1.5y = 90$    Equation 2

**Step 1:** Solve Equation 1 for $x$.

$\qquad x + y = 50$        Equation 1

$\qquad x = 50 - y$        Subtract $y$ from each side.

> **Study Tip**
>
> It is easiest to solve for a variable that has a coefficient of 1 or −1.

**Step 2:** Substitute $50 - y$ for $x$ in Equation 2.

$\qquad 2x + 1.5y = 90$            Equation 2

$\qquad 2(50 - y) + 1.5y = 90$     Substitute $50 - y$ for $x$.

$\qquad 100 - 2y + 1.5y = 90$      Distributive Property

$\qquad -0.5y = -10$               Simplify.

$\qquad y = 20$                    Divide each side by $-0.5$.

> **Check**
>
> 60
>
> 2x + 1.5y = 90
>
> x + y = 50
>
> Intersection
> X=30          Y=20          60
> 0

**Step 3:** Substitute 20 for $y$ in Equation 1 and solve for $x$.

$\qquad x + y = 50$        Equation 1

$\qquad x + 20 = 50$       Substitute 20 for $y$.

$\qquad x = 30$            Subtract 20 from each side.
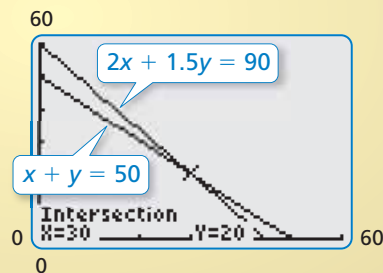
∴ You buy 30 turkey burgers and 20 veggie burgers.

### On Your Own

**4.** You sell lemonade for $2 per cup and orange juice for $3 per cup. You sell a total of 100 cups for $240. Write and solve a system of linear equations to find the number of cups of lemonade and the number of cups of orange juice you sold.

## ✓ Vocabulary and Concept Check

1. **WRITING** Describe how to solve a system of linear equations by substitution.

2. **NUMBER SENSE** When solving a system of linear equations by substitution, how do you decide which variable to solve for in Step 1?

3. **REASONING** Does solving a system of linear equations by graphing give the same solution as solving by substitution? Explain your reasoning.

## Practice and Problem Solving

**Write a system of linear equations that has the ordered pair as its solution. Use a method from Activity 1 to solve the system.**

4. 

5. 

6. 

**Tell which equation you would choose to solve for one of the variables when solving the system by substitution. Explain your reasoning.**

7. $2x + 3y = 5$
   $4x - y = 3$

8. $\frac{2}{3}x + 5y = -1$
   $x + 6y = 0$

9. $2x + 10y = 14$
   $5x - 9y = 1$

**Solve the system of linear equations by substitution. Check your solution.**

① 10. $y = x - 4$
   $y = 4x - 10$

11. $y = 2x + 5$
   $y = 3x - 1$

12. $x = 2y + 7$
   $3x - 2y = 3$

13. $4x - 2y = 14$
   $y = \frac{1}{2}x - 1$

14. $2x = y - 10$
   $x + 7 = y$

15. $8x - \frac{1}{3}y = 0$
   $12x + 3 = y$

16. **SCHOOL CLUBS** There are a total of 64 students in a drama club and a yearbook club. The drama club has 10 more students than the yearbook club.

   a. Write a system of linear equations that represents this situation.

   b. How many students are in the drama club? the yearbook club?

17. **THEATER** A drama club earns $1040 from a production. It sells a total of 64 adult tickets and 132 student tickets. An adult ticket costs twice as much as a student ticket.

   a. Write a system of linear equations that represents this situation.

   b. What is the cost of each ticket?

**Solve the system of linear equations by substitution. Check your solution.**

**② 18.** $y - x = 0$
$2x - 5y = 9$

**19.** $x + 4y = 14$
$3x + 7y = 22$

**20.** $-2x - 5y = 3$
$3x + 8y = -6$

**21. ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

✗  $2x + y = 5$  Equation 1
$3x - 2y = 4$  Equation 2

Step 1:
$2x + y = 5$
$y = -2x + 5$

Step 2:
$2x + (-2x + 5) = 5$
$2x - 2x + 5 = 5$
$5 = 5$

**22. STRUCTURE** The measure of the obtuse angle in the isosceles triangle is two and a half times the measure of one base angle. Write and solve a system of linear equations to find the measures of all the angles.
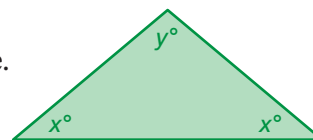
**23. ANIMAL SHELTER** An animal shelter has a total of 65 abandoned cats and dogs. The ratio of cats to dogs is 6 : 7. How many cats are in the shelter? How many dogs are in the shelter? Justify your answers.

**24. NUMBER SENSE** The sum of the digits of a two-digit number is 8. When the digits are reversed, the number increases by 36. Find the original number.

**25.** *Repeated Reasoning* A DJ has a total of 1075 dance, rock, and country songs on her system. The dance selection is three times the size of the rock selection. The country selection has 105 more songs than the rock selection. How many songs on the system are dance? rock? country?
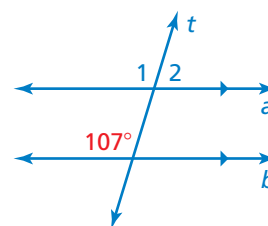
## Fair Game Review  *What you learned in previous grades & lessons*

**Write the equation in standard form.**  *(Section 4.5)*
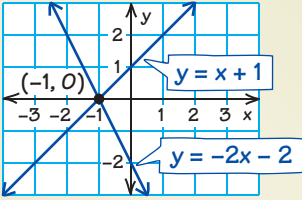
**26.** $3x - 9 = 7y$

**27.** $8 - 5y = -2x$

**28.** $6x = y + 3$

**29. MULTIPLE CHOICE** Use the figure to find the measure of $\angle 2$.  *(Section 3.1)*

Ⓐ  17°

Ⓑ  73°

Ⓒ  83°

Ⓓ  107°

**Check It Out**
Graphic Organizer
BigIdeasMath.com

You can use a **notetaking organizer** to write notes, vocabulary, and questions about a topic. Here is an example of a notetaking organizer for solving systems of linear equations by graphing.

Write important vocabulary or formulas in this space.

system of linear equations

solution of a system of linear equations

## Solving systems of linear equations by graphing

**Step 1:** Graph each equation.

**Step 2:** Estimate the point of intersection.

**Step 3:** Check the point from Step 2.

Example:
Solve the system. $y = x + 1$
$y = -2x - 2$



$y = x + 1$

$(-1, 0)$

$y = -2x - 2$

The solution is $(-1, 0)$.

Write your notes about the topic in this space.

Write your questions about the topic in this space.

Will a system of linear equations always have a solution?

## On Your Own

**Make a notetaking organizer to help you study this topic.**

1. solving systems of linear equations by substitution

**After you complete this chapter, make notetaking organizers for the following topics.**

2. solving systems of linear equations by elimination

3. solving systems of linear equations with no solution or infinitely many solutions

4. solving linear equations by graphing



Formulas:
Human Dog
1 yr = 7 yr

Notes:
Birthdays are 7, 14, 21, ...

Questions: Ask owner about fetching sticks

Aw ...the golden nonretriever years.

**"My notetaking organizer has me thinking about retirement when I won't have to fetch sticks anymore."**

**Check It Out**
Progress Check
BigIdeasMath ✓com

**Match the system of linear equations with the corresponding graph. Use the graph to estimate the solution. Check your solution.**    *(Section 5.1)*
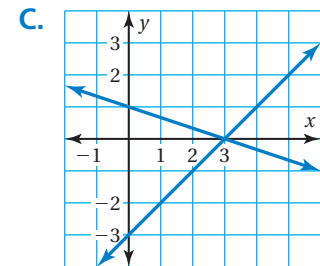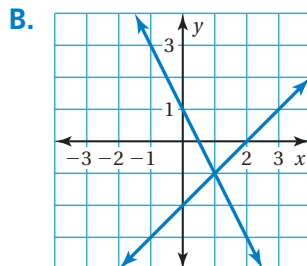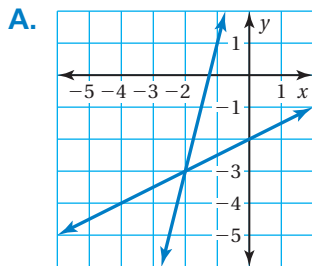
**1.** $y = x - 2$

$y = -2x + 1$

**2.** $y = x - 3$

$y = -\frac{1}{3}x + 1$

**3.** $y = \frac{1}{2}x - 2$

$y = 4x + 5$

**A.**

**B.**

**C.**

**Solve the system of linear equations by graphing.**    *(Section 5.1)*

**4.** $y = 2x - 3$
$y = -x + 9$

**5.** $6x + y = -2$
$y = -3x + 1$

**6.** $4x + 2y = 2$
$3x = 4 - y$

**Solve the system of linear equations by substitution. Check your solution.**
*(Section 5.2)*

**7.** $y = x - 8$
$y = 2x - 14$

**8.** $x = 2y + 2$
$2x - 5y = 1$

**9.** $x - 5y = 1$
$-2x + 9y = -1$

**10. MOVIE CLUB**  Members of a movie rental club pay a $15 annual membership fee and $2 for new release movies. Nonmembers pay $3 for new release movies.  *(Section 5.1)*

   **a.** Write a system of linear equations that represents this situation.

   **b.** When is it beneficial to have a membership?

**11. NUMBER SENSE**  The sum of two numbers is 38. The greater number is 8 more than the other number. Find each number. Use a system of linear equations to justify your answer.  *(Section 5.1)*

**12. VOLLEYBALL**  The length of a sand volleyball court is twice its width. The perimeter is 180 feet. Find the length and width of the sand volleyball court.  *(Section 5.2)*

**13. MEDICAL STAFF**  A hospital employs a total of 77 nurses and doctors. The ratio of nurses to doctors is 9 : 2. How many nurses are employed at the hospital? How many doctors are employed at the hospital?  *(Section 5.2)*

# Solving Systems of Linear Equations by Elimination

## Essential Question How can you use elimination to solve a system of linear equations?

### 1 ACTIVITY: Using Elimination to Solve a System
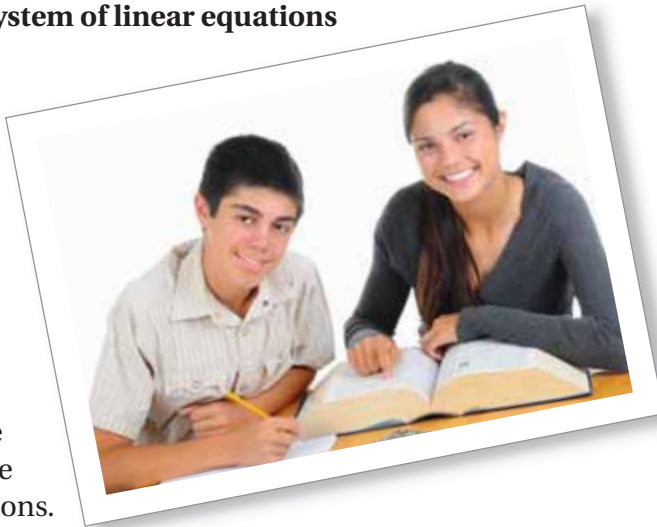
**Work with a partner. Solve each system of linear equations by using two methods.**

**Method 1:** Subtract.

Subtract Equation 2 from Equation 1. What is the result? Explain how you can use the result to solve the system of equations.

**Method 2:** Add.

Add the two equations. What is the result? Explain how you can use the result to solve the system of equations.

Is the solution the same using both methods?

**a.** $2x + y = 4$
$2x - y = 0$

**b.** $3x - y = 4$
$3x + y = 2$

**c.** $x + 2y = 7$
$x - 2y = -5$

### 2 ACTIVITY: Using Elimination to Solve a System

**Work with a partner.**

$2x + y = 2$   Equation 1
$x + 5y = 1$   Equation 2

**a.** Can you add or subtract the equations to solve the system of linear equations? Explain.

**b.** Explain what property you can apply to Equation 1 in the system so that the $y$-coefficients are the same.

**c.** Explain what property you can apply to Equation 2 in the system so that the $x$-coefficients are the same.

**d.** You solve the system in part (b). Your partner solves the system in part (c). Compare your solutions.

**e.** Use a graphing calculator to check your solution.

**COMMON CORE**

**Systems of Equations**
In this lesson, you will
- write and solve systems of linear equations by elimination.
- solve real-life problems.
Learning Standards
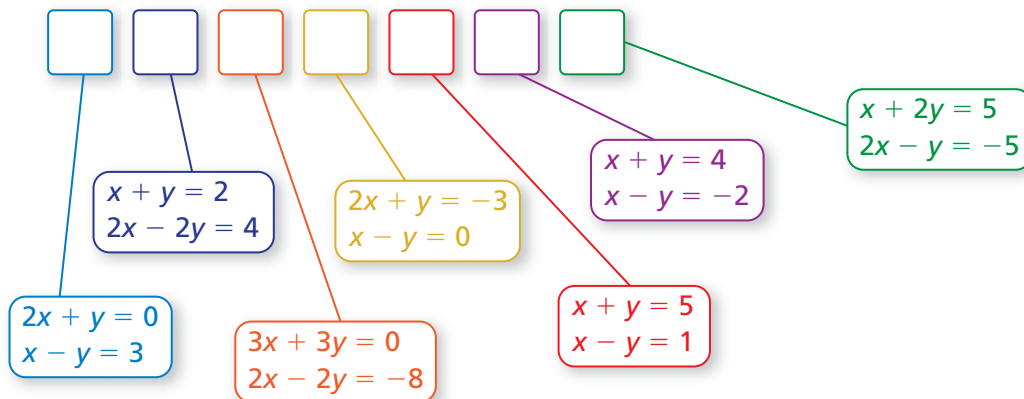8.EE.8b
8.EE.8c

**Math Practice** 1

**Find Entry Points**

What is the first thing you do to solve a system of linear equations? Why?

**Work with a partner. Solve the puzzle to find the name of a famous mathematician who lived in Egypt around 350 A.D.**

| | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| **4** | B | W | R | M | F | Y | K | N |
| **3** | O | J | A | S | I | D | X | Z |
| **2** | Q | P | C | E | G | B | T | J |
| **1** | M | R | C | Z | N | O | U | W |
| **0** | K | X | U | H | L | Y | S | Q |
| **−1** | F | E | A | S | W | K | R | M |
| **−2** | G | J | Z | N | H | V | D | G |
| **−3** | E | L | X | L | F | Q | O | B |

$x + 2y = 5$
$2x - y = -5$

$x + y = 4$
$x - y = -2$

$x + y = 2$
$2x - 2y = 4$

$2x + y = -3$
$x - y = 0$

$2x + y = 0$
$x - y = 3$

$3x + 3y = 0$
$2x - 2y = -8$

$x + y = 5$
$x - y = 1$

## What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use elimination to solve a system of linear equations?

5. **STRUCTURE** When can you add or subtract equations in a system to solve the system? When do you have to multiply first? Justify your answers with examples.

6. **LOGIC** In Activity 2, why can you multiply equations in the system by a constant and not change the solution of the system? Explain your reasoning.

**Practice**  Use what you learned about systems of linear equations to complete Exercises 4–6 on page 221.

## Key Idea

**Solving a System of Linear Equations by Elimination**

**Step 1:**  Multiply, if necessary, one or both equations by a constant so at least 1 pair of like terms has the same or opposite coefficients.

**Step 2:**  Add or subtract the equations to eliminate one of the variables.

**Step 3:**  Solve the resulting equation for the remaining variable.

**Step 4:**  Substitute the value from Step 3 into one of the original equations and solve.

**EXAMPLE 1   Solving a System of Linear Equations by Elimination**

**Study Tip**

Because the coefficients of $x$ are the same, you can also solve the system by subtracting in Step 2.

$$x + 3y = -2$$
$$\underline{x - 3y = 16}$$
$$6y = -18$$

So, $y = -3$.

**Check**

Equation 1

$$x + 3y = -2$$
$$7 + 3(-3) \stackrel{?}{=} -2$$
$$-2 = -2 ✓$$

Equation 2

$$x - 3y = 16$$
$$7 - 3(-3) \stackrel{?}{=} 16$$
$$16 = 16 ✓$$

Solve the system by elimination.        $x + 3y = -2$     Equation 1

$x - 3y = 16$     Equation 2

**Step 1:**  The coefficients of the $y$-terms are already opposites.

**Step 2:**  Add the equations.

| | |
|---|---|
| $x + 3y = -2$ | Equation 1 |
| $\underline{x - 3y = \ 16}$ | Equation 2 |
| $2x \qquad = \ 14$ | Add the equations. |

**Step 3:**  Solve for $x$.

| | |
|---|---|
| $2x = 14$ | Equation from Step 2 |
| $x = 7$ | Divide each side by 2. |

**Step 4:**  Substitute 7 for $x$ in one of the original equations and solve for $y$.

| | |
|---|---|
| $x + 3y = -2$ | Equation 1 |
| $7 + 3y = -2$ | Substitute 7 for $x$. |
| $3y = -9$ | Subtract 7 from each side. |
| $y = -3$ | Divide each side by 3. |

The solution is $(7, -3)$.

## On Your Own

**Now You're Ready**
Exercises 7–12

**Solve the system of linear equations by elimination. Check your solution.**

**1.**  $2x - y = 9$
     $4x + y = 21$

**2.**  $-5x + 2y = 13$
     $5x + y = -1$

**3.**  $3x + 4y = -6$
     $7x + 4y = -14$

**EXAMPLE** **2** **Solving a System of Linear Equations by Elimination**

**Solve the system by elimination.**    $-6x + 5y = 25$    Equation 1

$-2x - 4y = 14$    Equation 2

**Step 1:** Multiply Equation 2 by 3.

$-6x + 5y = 25$                              $-6x + 5y = 25$    Equation 1

$-2x - 4y = 14$  **Multiply by 3.**  ▶  $-6x - 12y = 42$    Revised Equation 2

**Step 2:** Subtract the equations.

$$\begin{array}{ll} -6x + 5y = 25 & \text{Equation 1} \\ -6x - 12y = 42 & \text{Revised Equation 2} \\ \hline \phantom{-6x+}17y = -17 & \text{Subtract the equations.} \end{array}$$

**Step 3:** Solve for $y$.

$17y = -17$    Equation from Step 2

$y = -1$    Divide each side by 17.

**Step 4:** Substitute $-1$ for $y$ in one of the original equations and solve for $x$.

$$\begin{array}{ll} -2x - 4y = 14 & \text{Equation 2} \\ -2x - 4(-1) = 14 & \text{Substitute } -1 \text{ for } y. \\ -2x + 4 = 14 & \text{Multiply.} \\ -2x = 10 & \text{Subtract 4 from each side.} \\ x = -5 & \text{Divide each side by } -2. \end{array}$$

∴  The solution is $(-5, -1)$.

**Check**
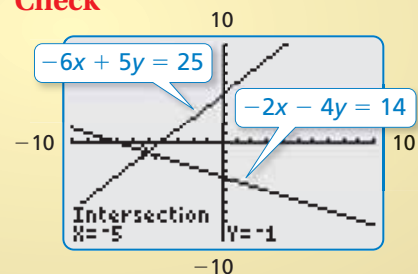


**On Your Own**

**Solve the system of linear equations by elimination. Check your solution.**

**4.**  $3x + y = 11$
$6x + 3y = 24$

**5.**  $4x - 5y = -19$
$-x - 2y = 8$

**6.**  $5y = 15 - 5x$
$y = -2x + 3$

EXAMPLE **3** — **Real-Life Application**

**You buy 8 hostas and 15 daylilies for $193. Your friend buys 3 hostas and 12 daylilies for $117. Write and solve a system of linear equations to find the cost of each daylily.**

Use a verbal model to write a system of linear equations.

| Number of hostas | · | Cost of each hosta, $x$ | + | Number of daylilies | · | Cost of each daylily, $y$ | = | Total cost |
|---|---|---|---|---|---|---|---|---|

The system is:   $8x + 15y = 193$     Equation 1 (You)
                 $3x + 12y = 117$     Equation 2 (Your friend)

**Step 1:** To find the cost $y$ of each daylily, eliminate the $x$-terms. Multiply Equation 1 by 3. Multiply Equation 2 by 8.

$8x + 15y = 193$   **Multiply by 3.** ➤   $24x + 45y = 579$     Revised Equation 1

$3x + 12y = 117$   **Multiply by 8.** ➤   $24x + 96y = 936$     Revised Equation 2

**Step 2:** Subtract the revised equations.

$$24x + 45y = \phantom{0}579 \qquad \text{Revised Equation 1}$$
$$\underline{24x + 96y = \phantom{0}936} \qquad \text{Revised Equation 2}$$
$$-51y = -357 \qquad \text{Subtract the equations.}$$

**Step 3:** Solving the equation $-51y = -357$ gives $y = 7$.

⋰   So, each daylily costs $7.

**On Your Own**

*Now You're Ready*
Exercises 16–21

**7.** A landscaper buys 4 peonies and 9 geraniums for $190. Another landscaper buys 5 peonies and 6 geraniums for $185. Write and solve a system of linear equations to find the cost of each peony.

## Summary

**Methods for Solving Systems of Linear Equations**

| Method | When to Use |
|---|---|
| Graphing *(Lesson 5.1)* | To estimate solutions |
| Substitution *(Lesson 5.2)* | When one of the variables in one of the equations has a coefficient of 1 or $-1$ |
| Elimination *(Lesson 5.3)* | When at least 1 pair of like terms has the same or opposite coefficients |
| Elimination (Multiply First) *(Lesson 5.3)* | When one of the variables cannot be eliminated by adding or subtracting the equations |

**Check It Out**
Help with Homework
**BigIdeasMath** com

## Vocabulary and Concept Check

1. **WRITING** Describe how to solve a system of linear equations by elimination.

2. **NUMBER SENSE** When should you use multiplication to solve a system of linear equations by elimination?

3. **WHICH ONE DOESN'T BELONG?** Which system of equations does *not* belong with the other three? Explain your reasoning.

| | | | |
|---|---|---|---|
| $3x + 3y = 3$ | $-2x + y = 6$ | $2x + 3y = 11$ | $x + y = 5$ |
| $2x - 3y = 7$ | $2x - 3y = -10$ | $3x - 2y = 10$ | $3x - y = 3$ |

## Practice and Problem Solving

**Use a method from Activity 1 to solve the system.**

4. $x + y = 3$
   $x - y = 1$

5. $-x + 3y = 0$
   $x + 3y = 12$

6. $3x + 2y = 3$
   $3x - 2y = -9$

**Solve the system of linear equations by elimination. Check your solution.**

① 7. $x + 3y = 5$
   $-x - y = -3$

8. $x - 2y = -7$
   $3x + 2y = 3$

9. $4x + 3y = -5$
   $-x + 3y = -10$

10. $2x + 7y = 1$
    $2x - 4y = 12$

11. $2x + 5y = 16$
    $3x - 5y = -1$

12. $3x - 2y = 4$
    $6x - 2y = -2$

13. **ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

❌ $5x + 2y = 9$  Equation 1
   $3x - 2y = -1$  Equation 2
   $\overline{2x \quad\quad = 10}$
   $x = 5$

The solution is $(5, -8)$.

14. **RAFFLE TICKETS** You and your friend are selling raffle tickets for a new laptop. You sell 14 more tickets than your friend sells. Together, you and your friend sell 58 tickets.

    a. Write a system of linear equations that represents this situation.
    b. How many tickets does each of you sell?

15. **JOGGING** You can jog around your block twice and the park once in 10 minutes. You can jog around your block twice and the park 3 times in 22 minutes.

    a. Write a system of linear equations that represents this situation.
    b. How long does it take you to jog around the park?

**Solve the system of linear equations by elimination. Check your solution.**

**(2) (3) 16.** $2x - y = 0$
$3x - 2y = -3$

**17.** $x + 4y = 1$
$3x + 5y = 10$

**18.** $-2x + 3y = 7$
$5x + 8y = -2$

**19.** $3x + 3 = 3y$
$2x - 6y = 2$

**20.** $2x - 6 = 4y$
$7y = -3x + 9$

**21.** $5x = 4y + 8$
$3y = 3x - 3$

**22. ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

✗ 
| $x + y = 1$ | Equation 1 | Multiply by $-5$. | $-5x + 5y = -5$ |
|---|---|---|---|
| $5x + 3y = -3$ | Equation 2 | | $\underline{5x + 3y = -3}$ |

$$8y = -8$$
$$y = -1$$

The solution is $(2, -1)$.

**23. REASONING** For what values of $a$ and $b$ should you solve the system by elimination?

**a.** $4x - y = 3$
$ax + 10y = 6$

**b.** $x - 7y = 6$
$-6x + by = 9$

**Determine whether the line through the first pair of points intersects the line through the second pair of points. Explain.**

**24.** Line 1: $(-2, 1)$, $(2, 7)$
Line 2: $(-4, -1)$, $(0, 5)$

**25.** Line 1: $(3, -2)$, $(7, -1)$
Line 2: $(5, 2)$, $(6, -2)$

**26. AIRPLANES** Two airplanes are flying to the same airport. Their positions are shown in the graph. Write a system of linear equations that represents this situation. Solve the system by elimination to justify your answer.
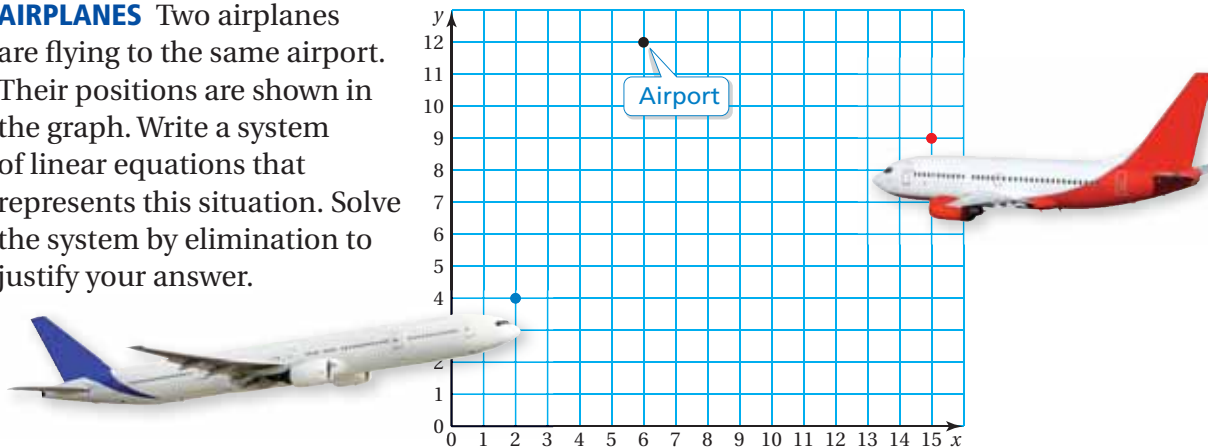


**27. TEST PRACTICE** The table shows the number of correct answers on a practice standardized test. You score 86 points on the test, and your friend scores 76 points.

| | You | Your Friend |
|---|---|---|
| **Multiple Choice** | 23 | 28 |
| **Short Response** | 10 | 5 |

**a.** Write a system of linear equations that represents this situation.

**b.** How many points is each type of question worth?

**28. LOGIC** You solve a system of equations in which $x$ represents the number of adult tickets sold and $y$ represents the number of student tickets sold. Can $(-6, 24)$ be the solution of the system? Explain your reasoning.

**29. VACATION** The table shows the activities of two tourists at a vacation resort. You want to go parasailing for 1 hour and horseback riding for 2 hours. How much do you expect to pay?

|  | Parasailing | Horseback Riding | Total Cost |
|---|---|---|---|
| **Tourist 1** | 2 hours | 5 hours | $205 |
| **Tourist 2** | 3 hours | 3 hours | $240 |

**30. REASONING** The solution of a system of linear equations is $(2, -4)$. One equation in the system is $2x + y = 0$. Explain how you could find a second equation for the system. Then find a second equation. Solve the system by elimination to justify your answer.

**31. JEWELER** A metal alloy is a mixture of two or more metals. A jeweler wants to make 8 grams of 18-carat gold, which is 75% gold. The jeweler has an alloy that is 90% gold and an alloy that is 50% gold. How much of each alloy should the jeweler use?

**32. PROBLEM SOLVING** A powerboat takes 30 minutes to travel 10 miles downstream. The return trip takes 50 minutes. What is the speed of the current?

**33.** **Critical Thinking** Solve the system of equations by elimination.

$$2x - y + 3z = -1$$
$$x + 2y - 4z = -1$$
$$y - 2z = 0$$

## Fair Game Review  What you learned in previous grades & lessons

**Decide whether the two equations are equivalent.**  *(Section 1.2 and Section 1.3)*

**34.** $4n + 1 = n - 8$

$3n = -9$

**35.** $2a + 6 = 12$

$a + 3 = 6$

**36.** $7v - \dfrac{3}{2} = 5$

$14v - 3 = 15$

**37. MULTIPLE CHOICE** Which line has the same slope as $y = \dfrac{1}{2}x - 3$?  *(Section 4.4)*

Ⓐ $y = -2x + 4$    Ⓑ $y = 2x + 3$    Ⓒ $y - 2x = 5$    Ⓓ $2y - x = 7$

## 5.4 Solving Special Systems of Linear Equations

**Essential Question** Can a system of linear equations have no solution? Can a system of linear equations have many solutions?

### 1 ACTIVITY: Writing a System of Linear Equations

**Work with a partner. Your cousin is 3 years older than you. You can represent your ages by two linear equations.**

$$y = t \qquad \text{Your age}$$
$$y = t + 3 \qquad \text{Your cousin's age}$$

a. Graph both equations in the same coordinate plane.

b. What is the vertical distance between the two graphs? What does this distance represent?

c. Do the two graphs intersect? Explain what this means in terms of your age and your cousin's age.
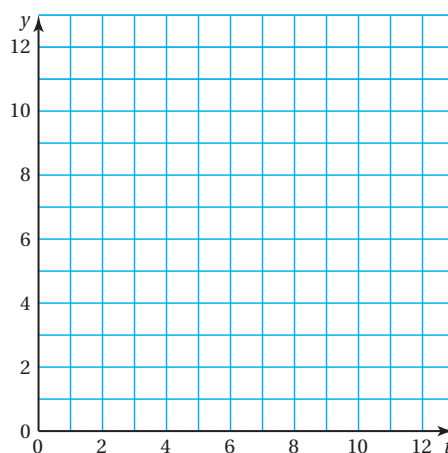
### 2 ACTIVITY: Using a Table to Solve a System

**Work with a partner. You invest $500 for equipment to make dog backpacks. Each backpack costs you $15 for materials. You sell each backpack for $15.**

a. Copy and complete the table for your cost $C$ and your revenue $R$.

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|----|
| C |   |   |   |   |   |   |   |   |   |   |    |
| R |   |   |   |   |   |   |   |   |   |   |    |

b. When will you break even? What is wrong?

**Math Practice 4**

**Analyze Relationships**

What do you know about the graphs of the two equations? How does this relate to the number of solutions?

**Work with a partner. Let $x$ and $y$ be two numbers. Here are two clues about the values of $x$ and $y$.**

| | **Words** | **Equation** |
|---|---|---|
| **Clue 1:** | $y$ is 4 more than twice the value of $x$. | $y = 2x + 4$ |
| **Clue 2:** | The difference of $3y$ and $6x$ is 12. | $3y - 6x = 12$ |

**a.** Graph both equations in the same coordinate plane.

**b.** Do the two lines intersect? Explain.

**c.** What is the solution of the puzzle?

**d.** Use the equation $y = 2x + 4$ to complete the table.
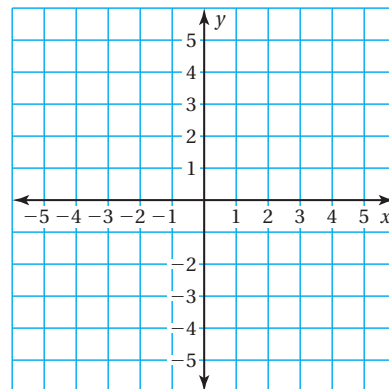


| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | | | | |

**e.** Does each solution in the table satisfy *both* clues?

**f.** What can you conclude? How many solutions does the puzzle have? How can you describe them?

## What Is Your Answer?

**4. IN YOUR OWN WORDS** Can a system of linear equations have no solution? Can a system of linear equations have many solutions? Give examples to support your answers.
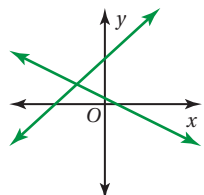
**Practice**

Use what you learned about special systems of linear equations to complete Exercises 3 and 4 on page 228.
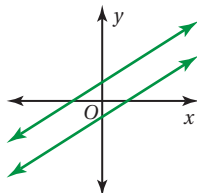
## 5.4 Lesson

### Key Idea

**Solutions of Systems of Linear Equations**

A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.
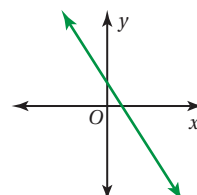


**One solution**
The lines intersect.

**No solution**
The lines are parallel.

**Infinitely many solutions**
The lines are the same.

---

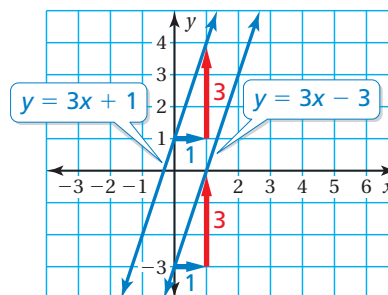### EXAMPLE 1  Solving a System: No Solution

**Solve the system.**   $y = 3x + 1$   Equation 1

$y = 3x - 3$   Equation 2

**Method 1:** Solve by graphing.

Graph each equation. The lines have the same slope and different $y$-intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.



**Study Tip**

You can solve some linear systems by inspection. In Example 1, notice you can rewrite the system as

$-3x + y = 1$
$-3x + y = -3$.

This system has no solution because $-3x + y$ cannot be equal to 1 and $-3$ at the same time.

So, the system of linear equations has no solution.

**Method 2:** Solve by substitution.

Substitute $3x - 3$ for $y$ in Equation 1.

$y = 3x + 1$   Equation 1

$3x - 3 = 3x + 1$   Substitute $3x - 3$ for $y$.

$-3 = 1$ ✗   Subtract $3x$ from each side.

The equation $-3 = 1$ is never true. So, the system of linear equations has no solution.

### On Your Own

**Now You're Ready**
Exercises 8–10

**Solve the system of linear equations. Check your solution.**

**1.** $y = -x + 3$
$y = -x + 5$

**2.** $y = -5x - 2$
$5x + y = 0$

**3.** $x = 2y + 10$
$2x + 3y = -1$

**EXAMPLE** **2**  **Solving a System: Infinitely Many Solutions**

**Rectangle A**

4y

2x

**Rectangle B**

12y

6x
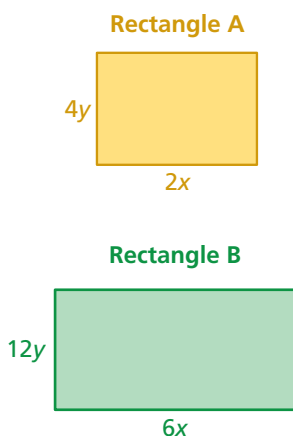
The perimeter of Rectangle A is 36 units. The perimeter of Rectangle B is 108 units. Write and solve a system of linear equations to find the values of $x$ and $y$.

*Perimeter of Rectangle A*

$$2(2x) + 2(4y) = 36$$
$$4x + 8y = 36 \quad \text{Equation 1}$$

*Perimeter of Rectangle B*

$$2(6x) + 2(12y) = 108$$
$$12x + 24y = 108 \quad \text{Equation 2}$$

The system is:  $4x + 8y = 36$     Equation 1
                $12x + 24y = 108$     Equation 2

**Method 1:** Solve by graphing.

Graph each equation.

The lines have the same slope and the same $y$-intercept. So, the lines are the same.

∵ In this context, $x$ and $y$ must be positive. Because the lines are the same, all the points on the line in Quadrant I are solutions of both equations. So, the system of linear equations has infinitely many solutions.
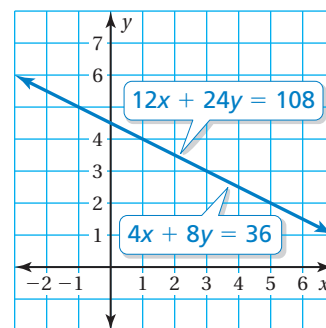
**Method 2:** Solve by elimination.

Multiply Equation 1 by 3 and subtract the equations.

$4x + 8y = 36$   **Multiply by 3.**   $12x + 24y = 108$     Revised Equation 1

$12x + 24y = 108$                    $\underline{12x + 24y = 108}$     Equation 2

                                        $0 = 0$     Subtract.

∵ The equation $0 = 0$ is always true. In this context, $x$ and $y$ must be positive. So, the solutions are all the points on the line $4x + 8y = 36$ in Quadrant I. The system of linear equations has infinitely many solutions.

## On Your Own

**Now You're Ready**
Exercises 11–13

Solve the system of linear equations. Check your solution.

**4.**   $x + y = 3$
     $x - y = -3$

**5.**   $2x + y = 5$
     $4x + 2y = 0$

**6.**   $2x - 4y = 10$
     $-12x + 24y = -60$

**7.**   **WHAT IF?** What happens to the solution in Example 2 if the perimeter of Rectangle A is 54 units? Explain.

**Check It Out**
Help with Homework
BigIdeasMath.com

## ✓ Vocabulary and Concept Check

1. **WRITING** Describe the difference between the graph of a system of linear equations that has *no solution* and the graph of a system of linear equations that has *infinitely many solutions*.

2. **REASONING** When solving a system of linear equations algebraically, how do you know when the system has *no solution*? *infinitely many solutions*?

## Practice and Problem Solving

**Let $x$ and $y$ be two numbers. Find the solution of the puzzle.**

3.
> $y$ is $\dfrac{1}{3}$ more than 4 times the value of $x$.
>
> The difference of $3y$ and $12x$ is 1.

4.
> $\dfrac{1}{2}$ of $x$ plus 3 is equal to $y$.
>
> $x$ is 6 more than twice the value of $y$.

**Without graphing, determine whether the system of linear equations has *one solution, infinitely many solutions,* or *no solution*. Explain your reasoning.**

5.  $y = 5x - 9$
    $y = 5x + 9$

6.  $y = 6x + 2$
    $y = 3x + 1$

7.  $y = 8x - 2$
    $y - 8x = -2$

**Solve the system of linear equations. Check your solution.**

① 8.  $y = 2x - 2$
    $y = 2x + 9$

9.  $y = 3x + 1$
    $-x + 2y = -3$

10. $y = \dfrac{\pi}{3}x + \pi$
    $-\pi x + 3y = -6\pi$

② 11. $y = -\dfrac{1}{6}x + 5$
    $x + 6y = 30$

12. $\dfrac{1}{3}x + y = 1$
    $2x + 6y = 6$

13. $-2x + y = 1.3$
    $2(0.5x - y) = 4.6$

14. **ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

> ✗   $y = -2x + 4$
>     $y = -2x + 6$
>
> The lines have the same slope, so, there are infinitely many solutions.

15. **PIG RACE** In a pig race, your pig gets a head start of 3 feet and is running at a rate of 2 feet per second. Your friend's pig is also running at a rate of 2 feet per second. A system of linear equations that represents this situation is $y = 2x + 3$ and $y = 2x$. Will your friend's pig catch up to your pig? Explain.

**16. REASONING** One equation in a system of linear equations has a slope of $-3$. The other equation has a slope of 4. How many solutions does the system have? Explain.

**17. LOGIC** How can you use the slopes and the $y$-intercepts of equations in a system of linear equations to determine whether the system has *one solution, infinitely many solutions,* or *no solution*? Explain your reasoning.

$$4x + 8y = 64$$
$$8x + 16y = 128$$

**18. MONEY** You and a friend both work two different jobs. The system of linear equations represents the total earnings for $x$ hours worked at the first job and $y$ hours worked at the second job. Your friend earns twice as much as you.

    **a.** One week, both of you work 4 hours at the first job. How many hours do you and your friend work at the second job?

    **b.** Both of you work the same number of hours at the second job. Compare the number of hours each of you works at the first job.

**19. DOWNLOADS** You download a digital album for $10. Then you and your friend download the same number of individual songs for $0.99 each. Write a system of linear equations that represents this situation. Will you and your friend spend the same amount of money? Explain.

**20. REASONING** Does the system shown *always, sometimes,* or *never* have no solution when $a = b$? $a \geq b$? $a < b$? Explain your reasoning.

$$y = ax + 1$$
$$y = bx + 4$$

**21. SKIING** The table shows the number of lift tickets and ski rentals sold to two different groups. Is it possible to determine how much each lift ticket costs? Justify your answer.

| Group | 1 | 2 |
|---|---|---|
| Number of Lift Tickets | 36 | 24 |
| Number of Ski Rentals | 18 | 12 |
| Total Cost (dollars) | 684 | 456 |

**22. Precision** Find the values of $a$ and $b$ so the system shown has the solution (2, 3). Does the system have any other solutions? Explain.

$$12x - 2by = 12$$
$$3ax - by = 6$$

## Fair Game Review  *What you learned in previous grades & lessons*

**Write an equation of the line that passes through the given points.**  *(Section 4.7)*

**23.** (0, 0), (2, 6)         **24.** (0, $-3$), (3, 3)         **25.** ($-6$, 5), (0, 2)

**26. MULTIPLE CHOICE** What is the solution of $-2(y + 5) \leq 16$?  *(Skills Review Handbook)*

    **A** $y \leq -13$       **B** $y \geq -13$       **C** $y \leq -3$       **D** $y \geq -3$

## 🔑 Key Idea

**COMMON CORE**

**Systems of Equations**
In this extension, you will
- solve linear equations by graphing a system of linear equations.
- solve real-life problems.

Applying Standards
8.EE.7
8.EE.8

**Solving Equations Using Graphs**

**Step 1:** To solve the equation $ax + b = cx + d$, write two linear equations.

$$ax + b = cx + d$$

$$y = ax + b \qquad \text{and} \qquad y = cx + d$$

**Step 2:** Graph the system of linear equations. The $x$-value of the solution of the system of linear equations is the solution of the equation $ax + b = cx + d$.

---

**EXAMPLE 1 Solving an Equation Using a Graph**

Solve $x - 2 = -\frac{1}{2}x + 1$ using a graph. Check your solution.

**Step 1:** Write a system of linear equations using each side of the equation.

$$x - 2 = -\frac{1}{2}x + 1$$

$$y = x - 2 \qquad\qquad y = -\frac{1}{2}x + 1$$

**Check**

$$x - 2 = -\frac{1}{2}x + 1$$

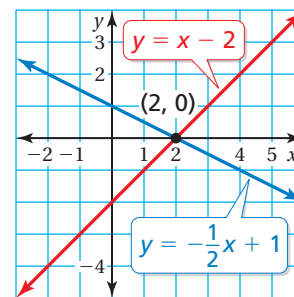$$2 - 2 \stackrel{?}{=} -\frac{1}{2}(2) + 1$$

$$0 = 0 \checkmark$$

**Step 2:** Graph the system.

$$y = x - 2$$

$$y = -\frac{1}{2}x + 1$$

The graphs intersect at $(2, 0)$.

So, the solution of the equation is $x = 2$.



## ● Practice

**Use a graph to solve the equation. Check your solution.**

1. $2x + 3 = 4$

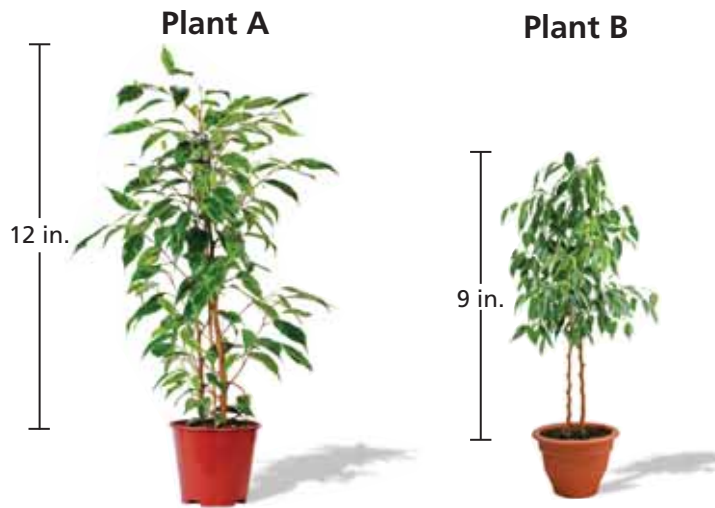2. $2x = x - 3$

3. $3x + 1 = 3x + 2$

4. $\frac{1}{3}x = x + 8$

5. $1.5x + 2 = 11 - 3x$

6. $3 - 2x = -2x + 3$

7. **STRUCTURE** Write an equation with variables on both sides that has no solution. How can you change the equation so that it has infinitely many solutions?

**EXAMPLE** **2** **Real-Life Application**

**Plant A**

**Plant B**

Plant A grows 0.6 inch per month. Plant B grows twice as fast.

12 in.

9 in.

a. Use the model to write an equation.

b. After how many months *x* are the plants the same height?

| Growth rate | • | Months, *x* | + | Original height | = | Growth rate | • | Months, *x* | + | Original height |
|---|---|---|---|---|---|---|---|---|---|---|

**a.** The equation is $0.6x + 12 = 1.2x + 9$.

**b.** Write a system of linear equations using each side of the equation. Then use a graphing calculator to graph the system.
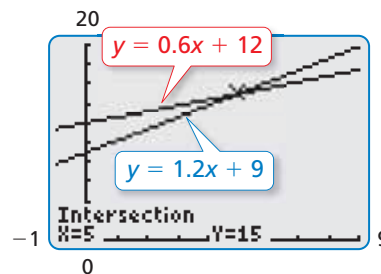
$$0.6x + 12 = 1.2x + 9$$

$y = 0.6x + 12$    $y = 1.2x + 9$

20

$y = 0.6x + 12$

$y = 1.2x + 9$

Intersection
X=5        Y=15        9

−1

0

The solution of the system is (5, 15).

So, the plants are both 15 inches tall after 5 months.

**Study Tip**

One way to check your answer is to solve the equation algebraically as in Section 1.3.

$0.6x + 12 = 1.2x + 9$
$12 = 0.6x + 9$
$3 = 0.6x$
$5 = x$

## Practice

**Use a graph to solve the equation. Check your solution.**

**8.** $6x - 2 = x + 11$

**9.** $\frac{4}{3}x - 1 = \frac{2}{3}x + 6$

**10.** $1.75x = 2.25x + 10.25$

**11.** **WHAT IF?** In Example 2, the growth rate of Plant A is 0.5 inch per month. After how many months *x* are the plants the same height?

**Solve the system of linear equations by elimination. Check your solution.** *(Section 5.3)*

**1.** $x + 2y = 4$
$-x - y = 2$

**2.** $2x - y = 1$
$x + 3y - 4 = 0$

**3.** $3x = -4y + 10$
$4x + 3y = 11$

**4.** $2x + 5y = 60$
$2x - 5y = -20$

**Solve the system of linear equations. Check your solution.** *(Section 5.4)*

**5.** $3x - 2y = 16$
$6x - 4y = 32$

**6.** $4y = x - 8$
$-\dfrac{1}{4}x + y = -1$

**7.** $-2x + y = -2$
$3x + y = 3$

**8.** $3x = \dfrac{1}{3}y + 2$
$9x - y = -6$

**Use a graph to solve the equation. Check your solution.** *(Section 5.4)*

**9.** $4x - 1 = 2x$

**10.** $-\dfrac{1}{2}x + 1 = -x + 1$

**11.** $1 - 3x = -3x + 2$

**12.** $1 - 5x = 3 - 7x$

**13. TOURS** The table shows the activities of two visitors at a park. You want to take the boat tour for 2 hours and the walking tour for 3 hours. Can you determine how much you will pay? Explain. *(Section 5.4)*

| | Boat Tour | Walking Tour | Total Cost |
|---|---|---|---|
| **Visitor 1** | 1 hour | 2 hours | $19 |
| **Visitor 2** | 1.5 hours | 3 hours | $28.50 |

**14. RENTALS** A business rents bicycles and in-line skates. Bicycle rentals cost $25 per day, and in-line skate rentals cost $20 per day. The business has 20 rentals today and makes $455. *(Section 5.3)*

   **a.** Write a system of linear equations that represents this situation.

   **b.** How many bicycle rentals and in-line skate rentals did the business have today?

# 5 Chapter Review

## Review Key Vocabulary

system of linear equations, *p. 204*          solution of a system of linear equations, *p. 204*

## Review Examples and Exercises

### 5.1 Solving Systems of Linear Equations by Graphing *(pp. 202–207)*

**Solve the system by graphing.**     $y = -2x$     Equation 1

$y = 3x + 5$     Equation 2

**Step 1:** Graph each equation.

**Step 2:** Estimate the point of intersection. The graphs appear to intersect at $(-1, 2)$.

**Step 3:** Check the point from Step 2.

| Equation 1 | Equation 2 |
|---|---|
| $y = -2x$ | $y = 3x + 5$ |
| $2 \stackrel{?}{=} -2(-1)$ | $2 \stackrel{?}{=} 3(-1) + 5$ |
| $2 = 2$ ✓ | $2 = 2$ ✓ |
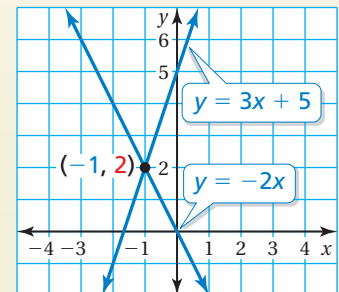
The solution is $(-1, 2)$.

#### Exercises

**Solve the system of linear equations by graphing.**

1. $y = 2x - 3$
   $y = x + 2$

2. $y = -x + 4$
   $x + 3y = 0$

3. $x - y = -2$
   $2x - 3y = -2$

### 5.2 Solving Systems of Linear Equations by Substitution *(pp. 208–213)*

**Solve the system by substitution.**     $x = 1 + y$     Equation 1

$x + 3y = 13$     Equation 2

**Step 1:** Equation 1 is already solved for $x$.

**Step 2:** Substitute $1 + y$ for $x$ in Equation 2.

$1 + y + 3y = 13$          Substitute $1 + y$ for $x$.

$y = 3$          Solve for $y$.

**Step 3:** Substituting 3 for $y$ in Equation 1 gives $x = 4$.

The solution is $(4, 3)$.

## Exercises

**Solve the system of linear equations by substitution. Check your solution.**

**4.** $y = -3x - 7$

$y = x + 9$

**5.** $\frac{1}{2}x + y = -4$

$y = 2x + 16$

**6.** $-x + 5y = 28$

$x + 3y = 20$

---

**5.3** **Solving Systems of Linear Equations by Elimination** *(pp. 216–223)*

You have a total of 5 quarters and dimes in your pocket. The value of the coins is $0.80. Write and solve a system of linear equations to find the number $x$ of dimes and the number $y$ of quarters in your pocket.

Use a verbal model to write a system of linear equations.

| Number of dimes, $x$ | $+$ | Number of quarters, $y$ | $=$ | Number of coins |

| Value of a dime | $\cdot$ | Number of dimes, $x$ | $+$ | Value of a quarter | $\cdot$ | Number of quarters, $y$ | $=$ | Total value |

The system is $x + y = 5$ and $0.1x + 0.25y = 0.8$.

**Step 1:** Multiply Equation 2 by 10.

$x + y = 5$        $x + y = 5$     Equation 1

$0.1x + 0.25y = 0.8$ **Multiply by 10.** $x + 2.5y = 8$     Revised Equation 2

**Step 2:** Subtract the equations.

$$\begin{array}{ll} x + \phantom{2.5}y = \phantom{-}5 & \text{Equation 1} \\ x + 2.5y = \phantom{-}8 & \text{Revised Equation 2} \\ \hline \phantom{x +}-1.5y = -3 & \text{Subtract the equations.} \end{array}$$

**Step 3:** Solving the equation $-1.5y = -3$ gives $y = 2$.

**Step 4:** Substitute 2 for $y$ in one of the original equations and solve for $x$.

$x + y = 5$         Equation 1

$x + 2 = 5$       Substitute 2 for $y$.

$x = 3$           Subtract 2 from each side.

So, you have 3 dimes and 2 quarters in your pocket.

---

## Exercises

**7. GIFT BASKET** A gift basket that contains jars of jam and packages of bread mix costs $45. There are 8 items in the basket. Jars of jam cost $6 each, and packages of bread mix cost $5 each. Write and solve a system of linear equations to find the number of jars of jam and the number of packages of bread mix in the gift basket.

## 5.4 Solving Special Systems of Linear Equations  *(pp. 224–231)*

**a.** **Solve the system.**  $y = -5x - 8$  Equation 1

$y = -5x + 4$  Equation 2

Solve by substitution. Substitute $-5x + 4$ for $y$ in Equation 1.

$$y = -5x - 8$$  Equation 1

$$-5x + 4 = -5x - 8$$  Substitute $-5x + 4$ for $y$.

$$4 = -8 \; ✗$$  Add $5x$ to each side.

The equation $4 = -8$ is never true. So, the system of linear equations has no solution.

**b.** **Solve** $x + 1 = \dfrac{1}{3}x + 3$ **using a graph. Check your solution.**

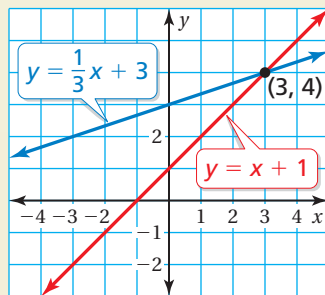**Step 1:** Write a system of linear equations using each side of the equation.

$$x + 1 = \frac{1}{3}x + 3$$

$$\boxed{y = x + 1} \qquad \boxed{y = \frac{1}{3}x + 3}$$

**Step 2:** Graph the system.

$$y = x + 1$$

$$y = \frac{1}{3}x + 3$$



**Check**

$$x + 1 = \frac{1}{3}x + 3$$

$$3 + 1 \overset{?}{=} \frac{1}{3}(3) + 3$$

$$4 = 4 \; ✔$$

The graphs intersect at (3, 4).

So, the solution is $x = 3$.

### Exercises

**Solve the system of linear equations. Check your solution.**

**8.** $x + 2y = -5$
$x - 2y = -5$

**9.** $3x - 2y = 1$
$9x - 6y = 3$

**10.** $8x - 2y = 16$
$-4x + y = 8$

**11.** Use a graph to solve $2x - 9 = 7x + 11$. Check your solution.

**Solve the system of linear equations by graphing.**

**1.** $y = 4 - x$

   $y = x - 4$

**2.** $y = \dfrac{1}{2}x + 10$

   $y = 4x - 4$

**3.** $y + x = 0$

   $3y + 6x = -9$

**Solve the system of linear equations by substitution. Check your solution.**

**4.** $-3x + y = 2$

   $-x + y - 4 = 0$

**5.** $x + y = 20$

   $y = 2x - 1$

**6.** $x - y = 3$

   $x + 2y = -6$

**Solve the system of linear equations by elimination. Check your solution.**

**7.** $2x + y = 3$

   $x - y = 3$

**8.** $x + y = 12$

   $3x = 2y + 6$

**9.** $-2x + y + 3 = 0$

   $3x + 4y = -1$

**Without graphing, determine whether the system of linear equations has
*one solution*, *infinitely many solutions*, or *no solution*. Explain your reasoning.**

**10.** $y = 4x + 8$

   $y = 5x + 1$

**11.** $2y = 16x - 2$

   $y = 8x - 1$

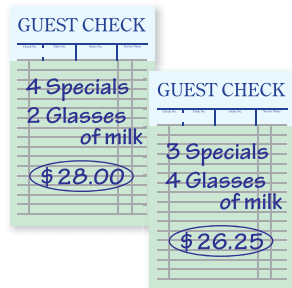**12.** $y = -3x + 2$

   $6x + 2y = 10$

**Use a graph to solve the equation. Check your solution.**

**13.** $\dfrac{1}{4}x - 4 = \dfrac{3}{4}x + 2$

**14.** $8x - 14 = -2x - 4$

**15. FRUIT** The price of 2 pears and 6 apples is $14. The price of 3 pears and 9 apples is $21. Can you determine the unit prices for pears and apples? Explain.

**16. BOUQUET** A bouquet of lilies and tulips has 12 flowers. Lilies cost $3 each, and tulips cost $2 each. The bouquet costs $32. Write and solve a system of linear equations to find the number of lilies and tulips in the bouquet.

GUEST CHECK

4 Specials
2 Glasses
   of milk

$28.00

GUEST CHECK

3 Specials
4 Glasses
   of milk

$26.25

**17. DINNER** How much does it cost for 2 specials and 2 glasses of milk?

1. What is the solution of the system of equations shown below? *(8.EE.8b)*

$$y = -\frac{2}{3}x - 1$$

$$4x + 6y = -6$$

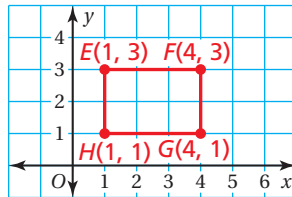**A.** $\left(-\frac{3}{2}, 0\right)$  **C.** no solution
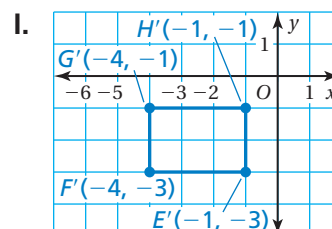
**B.** $(0, -1)$  **D.** infinitely many solutions

2. What is the slope of a line that is perpendicular to the line $y = -0.25x + 3$? *(8.EE.6)*

**Test-Taking Strategy**
**Read Question Before Answering**

I buy 2 toys and I new leash for $7. You buy 3 toys and I new leash for $9. How much is a new leash?

A leash? Me? I don't think so!

Ⓐ $2  Ⓑ $3  Ⓒ $4  Ⓓ $5

*"Take your time and read the question carefully before choosing your answer."*

3. On the grid below, Rectangle *EFGH* is plotted and its vertices are labeled.

E(1, 3)  F(4, 3)
H(1, 1)  G(4, 1)

Which of the following shows Rectangle $E'F'G'H'$, the image of Rectangle *EFGH* after it is reflected in the *x*-axis? *(8.G.3)*

**F.**
E'(1, −1)  F'(4, −1)
H'(1, −3)  G'(4, −3)

**H.**
F'(−4, 3) E'(−1, 3)
G'(−4, 1) H'(−1, 1)

**G.**
H'(1, −1)  G'(4, −1)
E'(1, −3)  F'(4, −3)

**I.**
H'(−1, −1)
G'(−4, −1)
F'(−4, −3)
E'(−1, −3)

**4.** Which point is a solution of the system of equations shown below? *(8.EE.8b)*

$$x + 3y = 10$$
$$x = 2y - 5$$

  **A.** $(1, 3)$                  **C.** $(55, -15)$

  **B.** $(3, 1)$                  **D.** $(-35, -15)$

**5.** The graph of a system of two linear equations is shown. How many solutions does the system have? *(8.EE.8a)*
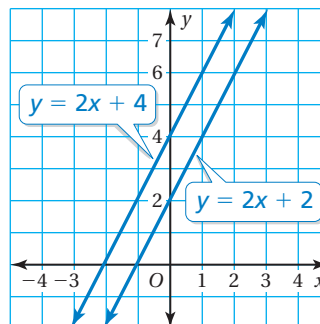
  **F.** none

  **G.** exactly one

  **H.** exactly two

  **I.** infinitely many



**6.** A scenic train ride has one price for adults and one price for children. One family of two adults and two children pays $62 for the train ride. Another family of one adult and four children pays $70. Which system of linear equations can you use to find the price $x$ for an adult and the price $y$ for a child? *(8.EE.8c)*

  **A.** $2x + 2y = 70$
       $x + 4y = 62$

  **C.** $2x + 2y = 62$
       $4x + y = 70$

  **B.** $x + y = 62$
       $x + y = 70$

  **D.** $2x + 2y = 62$
       $x + 4y = 70$

**7.** Which of the following is true about the graph of the linear equation $y = -7x + 5$? *(8.EE.6)*

  **F.** The slope is 5, and the $y$-intercept is $-7$.

  **G.** The slope is $-5$, and the $y$-intercept is $-7$.

  **H.** The slope is $-7$, and the $y$-intercept is $-5$.

  **I.** The slope is $-7$, and the $y$-intercept is 5.

**8.** What value of $w$ makes the equation below true? *(8.EE.7b)*

$$7w - 3w = 2(3w + 11)$$

**9.** The graph of which equation is parallel to the line that passes through the points $(-1, 5)$ and $(4, 7)$? *(8.EE.6)*

**A.** $y = \dfrac{2}{3}x + 6$

**C.** $y = \dfrac{2}{5}x + 1$

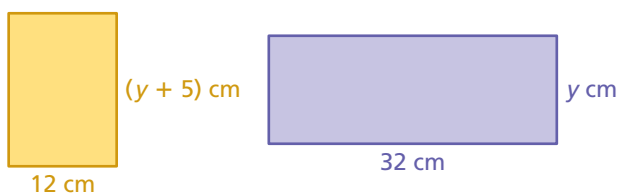**B.** $y = -\dfrac{5}{2}x + 4$

**D.** $y = \dfrac{5}{2}x - 1$

**10.** You buy 3 T-shirts and 2 pairs of shorts for $42.50. Your friend buys 5 T-shirts and 3 pairs of shorts for $67.50. Use a system of linear equations to find the cost of each T-shirt. Show your work and explain your reasoning. *(8.EE.8c)*

**Think**
**Solve**
**Explain**

**11.** The two figures have the same area. What is the value of $y$? *(8.EE.7b)*



$(y + 5)$ cm

$y$ cm

32 cm

12 cm

**F.** $\dfrac{1}{4}$

**H.** 3

**G.** $\dfrac{15}{8}$

**I.** 8

**12.** A system of two linear equations has infinitely many solutions. What can you conclude about the graphs of the two equations? *(8.EE.8a)*

**A.** The lines have the same slope and the same $y$-intercept.

**B.** The lines have the same slope and different $y$-intercepts.

**C.** The lines have different slopes and the same $y$-intercept.

**D.** The lines have different slopes and different $y$-intercepts.

**13.** The sum of one-third of a number and 10 is equal to 13. What is the number? *(8.EE.7b)*

**F.** $\dfrac{8}{3}$

**G.** 9

**H.** 29

**I.** 69

**14.** Solve the equation $4x + 7y = 16$ for $x$. *(8.EE.7b)*

**A.** $x = 4 + \dfrac{7}{4}y$

**C.** $x = 4 + \dfrac{4}{7}y$

**B.** $x = 4 - \dfrac{7}{4}y$

**D.** $x = 16 - 7y$